Correction to the Wing Source Velocity Error in Woodward's USSAERO Code

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All known versions of Woodward's USSAERO computer code suffer from a basic error which gives rise to incorrect vertical velocity components at field points away from the wing plane if the wing is tapered. This error can have serious consequences for external store load calculations. The problem is caused by incorrect analytical expressions in USSAERO for the vertical perturbation velocity component induced by the linear source distribution on a wing panel. Two different remedies are presented herein. The first remedy is a corrected version of the wing panel source distribution equations used in USSAERO. The other remedy is the use of a modified source distribution.

Nomenclature

$\bar{c}$	= panel chord at centroid line
d	= distance, Eq. (5)
F_1, G_1, G_2	= functions, Eqs. (9)
F_2, G_3, G_4	= functions, Eqs. (10)
K	= function, Eq. (3)
M_∞	= freestream Mach number
r	= function, Eq. (4)
UC, VC, WC	= velocity components induced by constant unit source distribution on semi-infinite region
UL, VL, WL	= velocity components induced by linear unit source distribution on semi-infinite region
u, v, w	= velocity components induced by source panel
V_∞	= freestream velocity
x, y, z	= Cartesian coordinates
α	= angle of attack
λ	= leading edge slope of semi-infinite region
ξ	= axial coordinate
τ	= local value of source distribution

Introduction

THE computer code USSAERO is used to compute the subsonic and supersonic potential flow aerodynamic characteristics of configurations. The body is represented by constant source panels and the lifting surfaces by linearly varying vortex panels and linearly varying source panels (planar boundary condition option). The original version of the USSAERO computer code, as developed by Woodward, was based on the theory described in Ref. 1. This version was written for wing/body/tail combinations. A later version was extended with a flowfield computation capability.[2] The author had the opportunity to experiment with that version several years ago and found that the vertical component of the flowfield below a tapered wing was very dependent on the chordwise wing paneling and was in fact incorrect for any chordwise paneling. The cause was traced back to incorrect analytical expressions for the vertical perturbation velocity component induced by the linear source distribution on a wing panel. Recently, the author viewed a listing of one of the latest USSAERO versions (version B01 with supersonic triplets[3]). It appears that the basic error mentioned above is still present, so that the calculation of the flowfield is still incorrect. The calculation of external store loads similarly becomes very unreliable.

Since the various versions of USSAERO are widely used all over the world, the present paper will demonstrate how to solve the above-mentioned problem.

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The Problem

Consider a tapered arrow wing with NACA 65A006 airfoil sections in the streamwise direction. The spanwise paneling is fixed (Fig. 1), but the chordwise paneling is varied. Two chordwise paneling cases are considered. In the first case there are 10 evenly distributed panels in the chordwise direction (panel edges at 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100% chord). In the second case there are 22 panels with higher panel density near the leading edge (panel edges at 0, 2.5, 5, 7.5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, and 100% chord). The flowfield perturbation velocities are calculated along an axial line l below the wing (Fig. 1). The results for $M_\infty = 0.4$, $\alpha = 6$ deg, are shown in Fig. 2 for the two different chordwise paneling cases, using the incorrect USSAERO code. The axial (u/V_∞) and lateral (v/V_∞) perturbation velocity components are seen to be practically independent of the paneling. The vertical component (w/V_∞) is, however, severely dependent on the chordwise paneling.

Mathematical Formulation

In USSAERO, constant source distributions and source distributions that vary linearly in chordwise direction are used to define linear source distributions on wing panels in a manner discussed below.

Consider an arbitrary wing panel. A semi-infinite region is associated with each corner point of the panel and is bounded by the panel edges at that corner point. The four semi-infinite regions lie in the plane of the panel. The velocity components at a point P induced by the actual source panel are then obtained by superposition of the velocity components at P induced by each of these four semi-infinite regions. The principle is illustrated in Fig. 3. Once the expressions for the velocity components are known for semi-infinite region 1 associated with panel corner point 1, the velocity components induced by the other three semi-infinite regions are readily obtained from these expressions by using the proper value of the edge slope and by shifting the origin to the corner under consideration.

In USSAERO, a linear source distribution on a panel is obtained as follows: The source distribution is taken as constant along lines parallel to the leading edge of each semi-infinite region. It starts from zero at the leading edge of each semi-infinite region and varies linearly in chordwise direction. The situation is illustrated in Fig. 4a. The panel source distribution is linear in the chordwise direction but, if the panel is tapered, it also varies linearly along a radial line, including the panel trailing edge (a radial line is a line through the intersection point of panel leading and trailing edge). It is seen from Fig. 4a that a constant source distribution extending downstream from the panel trailing edge must be subtracted to remove the source distribution behind the panel. The corresponding superposition principle for the velocity components u, v, and w at a point P induced by the above linear panel source distribution is then:

$$u = \{ (UL_1 - UL_2 - UL_3 + UL_4) - (UC_2 - UC_4) \}\tau_E \quad (1a)$$

$$v = \{ (VL_1 - VL_2 - VL_3 + VL_4) - (VC_2 - VC_4) \}\tau_E \quad (1b)$$

$$w = \{ (WL_1 - WL_2 - WL_3 + WL_4) - (WC_2 - WC_4) \}\tau_E \quad (1c)$$

where UC_i, VC_i, WC_i denote the velocity components at point P induced by the constant unit source distribution on semi-infinite region i. Similarly, UL_i, VL_i, WL_i denote the velocity components at P induced by the linearly varying source distribution on semi-infinite region i; the source distribution is such that it is unity at the panel trailing edge at the centroid line. Finally, τ_E denotes the value of the source distribution at the trailing edge at the centroid line of the panel.

Figure 4b shows that for a tapered panel, the source distribution behind the panel cannot be completely cancelled. As the source distribution varies along a radial line, a residual source distribution remains behind the panel; the distribution

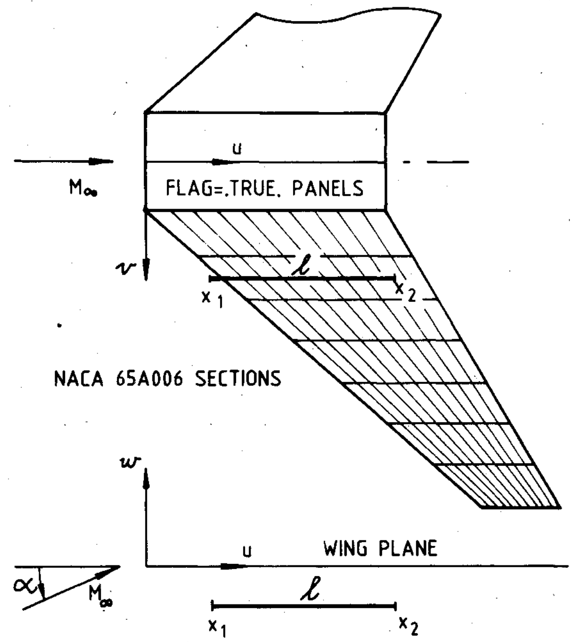

Fig. 1 Wing geometry, paneling, and line l.

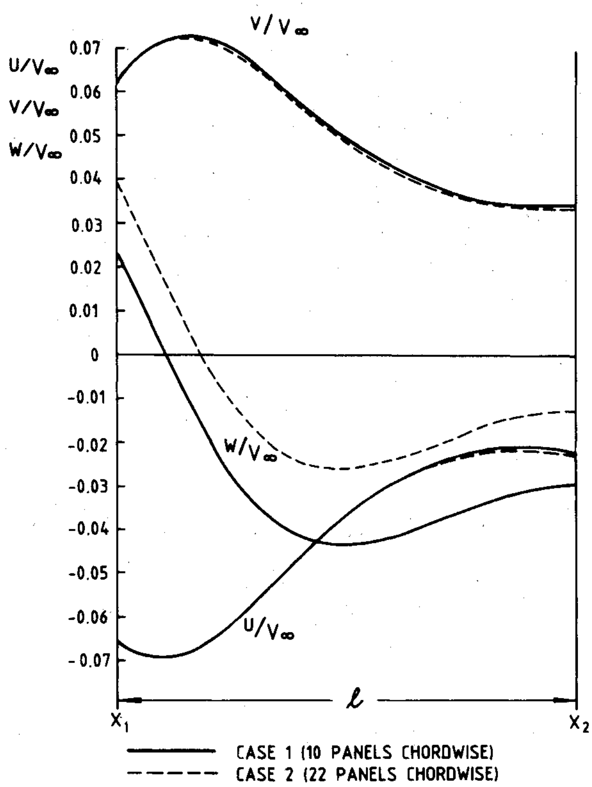

Fig. 2 The perturbation velocity components along line l using the incorrect USSAERO code ($M_\infty = 0.4$, $\alpha = 6$ deg).

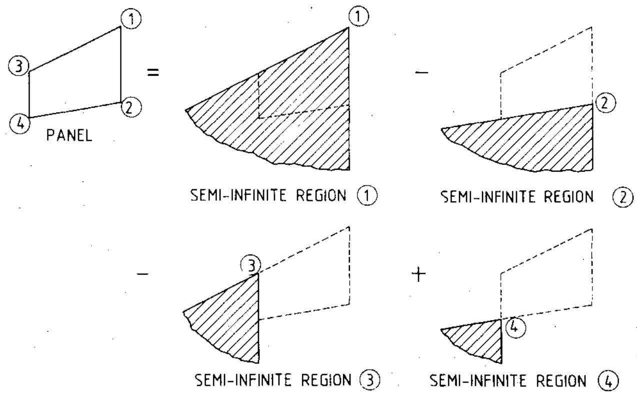

Fig. 3 Superposition principle for wing panel source distribution.

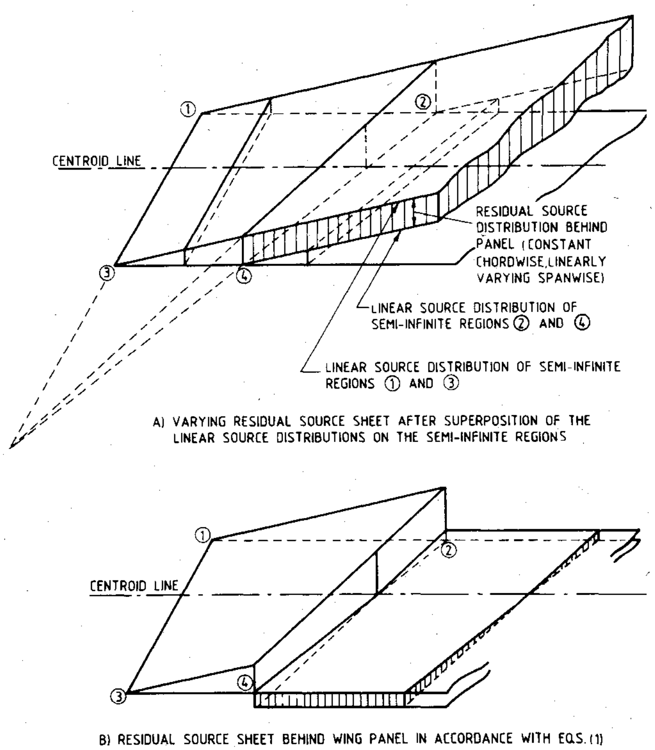

Fig. 4 Linear wing panel source distribution (linear along radial lines).

in this sheet is constant in chordwise direction and varies linearly in spanwise direction. Only along the centroid line is the strength of the residual source distribution zero, whereas it should be zero everywhere in the panel plane outside the panel.

The velocity components UC, VC, WC and UL, VL, WL will now be derived for an arbitrary semi-infinite region (consequently the subscripts will be omitted). Only the incompressible case is considered here, but the results hold equally well for the compressible case.

The distribution is constant along lines parallel to the leading edge of the semi-infinite region. In the case of a constant source distribution, its value is unity everywhere. The velocity components at a point P induced by this constant unit source distribution on the semi-infinite region are given by[1,4]:

$$UC=\frac{1}{4\pi}\frac{1}{1+\lambda^2}\left\{\int_0^\infty(x-\xi-\lambda y)K\mathrm{d}\xi-\lambda\int_0^\infty\frac{1}{d}\mathrm{d}\xi\right\} \tag{2a}$$

$$VC=-\frac{1}{4\pi}\frac{1}{1+\lambda^2}\left\{\lambda\int_0^\infty(x-\xi-\lambda y)K\mathrm{d}\xi+\int_0^\infty\frac{1}{d}\mathrm{d}\xi\right\} \tag{2b}$$

$$WC=\frac{z}{4\pi}\int_0^\infty K\mathrm{d}\xi \tag{2c}$$

where

$$K=\frac{\lambda(x-\xi-\lambda y)+(1+\lambda^2)y}{r^2 d} \tag{3}$$

$$r=\sqrt{(x-\xi-\lambda y)^2+(1+\lambda^2)z^2} \tag{4}$$

$$d=\sqrt{(x-\xi)^2+y^2+z^2} \tag{5}$$

The various variables are shown in Fig. 5. In the case of the linear source distribution, the distribution is zero along the leading edge ($\xi=0$) of the semi-infinite region and has a const value $\tau=\xi/\bar{c}$ along lines $\xi=$ const parallel to its leading edge, Fig. 5. Here, $\bar{c}$ denotes the panel chord at the centroid line. The velocity components at point P induced by this linear source distribution on the semi-infinite region are thus obtained by multiplying the integrands of Eqs. (2) with $\xi/\bar{c}$ prior to integration. Hence

$$UL=\frac{1}{4\pi\bar{c}}\frac{1}{1+\lambda^2}\left\{\int_0^\infty(x-\xi-\lambda y)K\xi\mathrm{d}\xi-\lambda\int_0^\infty\frac{1}{d}\xi\mathrm{d}\xi\right\} \tag{6a}$$

$$VL=-\frac{1}{4\pi\bar{c}}\frac{1}{1+\lambda^2}\left\{\lambda\int_0^\infty(x-\xi-\lambda y)K\xi\mathrm{d}\xi+\int_0^\infty\frac{1}{d}\xi\mathrm{d}\xi\right\} \tag{6b}$$

$$WL=\frac{z}{4\pi\bar{c}}\int_0^\infty K\xi\mathrm{d}\xi \tag{6c}$$

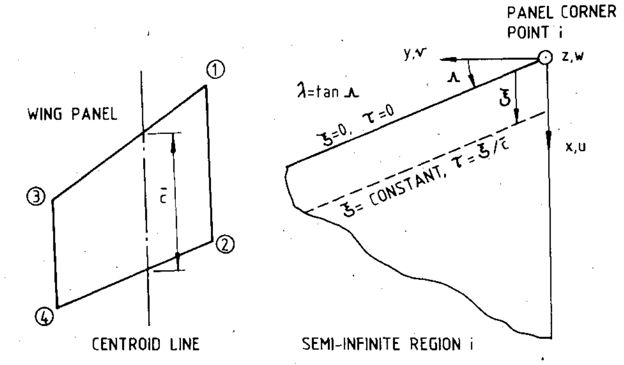

Fig. 5 Notation.

Solution

Solution of the integrals of Eqs. (2) and (6) gives

$$UC=-(1/4\pi)G_1 \tag{7a}$$

$$VC=(1/4\pi)\{\lambda G_1-(G_2-G_3)\} \tag{7b}$$

$$WC=(1/4\pi)(F_1-F_2) \tag{7c}$$

and

$$UL=(1/4\pi\bar{c})\{z(F_1-F_2)-(x-\lambda y)G_1-y(G_2-G_3)\} \tag{8a}$$

$$VL=(1/4\pi\bar{c})[(x-\lambda y)\{\lambda G_1-(G_2-G_3)\}+G_4+\sqrt{x^2+y^2+z^2}-\lambda z(F_1-F_2)] \tag{8b}$$

$$WL=(1/4\pi\bar{c})[(x-\lambda y)(F_1-F_2)+z\{(1+\lambda^2)G_1-\lambda(G_2-G_3)\}] \tag{8c}$$

where

$$F_1=\tan^{-1}\left(\frac{z\sqrt{x^2+y^2+z^2}}{\lambda(y^2+z^2)-yx}\right) \tag{9a}$$

$$G_1=\frac{1}{\sqrt{1+\lambda^2}}\ln\left(\frac{\lambda x+y+\sqrt{x^2+y^2+z^2}\sqrt{1+\lambda^2}}{\sqrt{(x-\lambda y)^2+(1+\lambda^2)z^2}}\right) \tag{9b}$$

$$G_2=\ln\left(\frac{x+\sqrt{x^2+y^2+z^2}}{\sqrt{y^2+z^2}}\right) \tag{9c}$$

These functions form the integral solutions for the limit $\xi=0$. The functions F_2, G_3, and G_4 are integral solutions for the limit $\xi=\infty$, viz

$$F_2=\lim_{\xi\to\infty}\tan^{-1}\left(\frac{zd}{\lambda(y^2+z^2)-y(x-\xi)}\right)$$

$$G_3=\lim_{\xi\to\infty}\ln\left(\frac{x-\xi+d}{\sqrt{y^2+z^2}}\right)$$

$$G_4=-\lim_{\xi\to\infty}d$$

[The limit for $\xi\to\infty$ associated with the function G_1 is cancelled in the superposition of semi-infinite regions 1 and 3 (or 2 and 4), and is therefore not included.]

Careful manipulation of the limits gives

$$F_2=\tan^{-1}(z/y) \tag{10a}$$

$$G_3=\ln\sqrt{y^2+z^2} \tag{10b}$$

$$G_4=x \tag{10c}$$

(The CDC Fortran IV external function ATAN2 should be used to be consistent with the USSAERO code.) Since the functions F_2, G_3, and G_4 were obtained at infinity downstream, they vanish in supersonic flow.

The function G_3 in the expression for VC and the function F_2 in the expression for WC would cancel if one superimposed semi-infinite regions 1 and 2 (y and z are the same for these regions). The same holds for semi-infinite regions 3 and 4. However, such a superposition does not take place; only the semi-infinite regions 2 and 4 are superimposed, see Eqs. (1). The functions G_3 and F_2 must therefore be retained in Eqs. (7).

For the linear source distribution, superposition of all four semi-infinite regions takes place, see Eqs. (1). Thus, the functions G_3 and F_2 could be omitted from the expression for UL. For the sake of conformity, these functions are, however, retained in the UL expression.

A detailed derivation of the above expressions is given in Ref. 4.

Results

The (incompressible) velocity components used in USSAERO are

$$UC = -(1/4\pi)G_1 \tag{11a}$$

$$VC = (1/4\pi)\{\lambda G_1 - (G_2 - G_3)\} \tag{11b}$$

$$WC = (1/4\pi)(F_1 - F_2) \tag{11c}$$

and

$$UL = (1/4\pi\bar{c})\{z(F_1 - F_2) - (x - \lambda y)G_1 - yG_2\} \tag{12a}$$

$$VL = (1/4\pi\bar{c})[(x-\lambda y)\{\lambda G_1 - (G_2 - G_3)\} + G_4 + \sqrt{x^2+y^2+z^2} - \lambda z(F_1 - F_2)] \tag{12b}$$

$$WL = (1/4\pi\bar{c})[(x-\lambda y)(F_1 - F_2) + z\{(1+\lambda^2)G_1 - \lambda G_2\}] \tag{12c}$$

By comparing Eqs. (11) and (12) with Eqs. (7) and (8), and recalling the comment regarding the function G_3 in the UL expression of Eqs. (8), it is observed that the expressions for UC, VC, WC, UL, and VL are the same. The expression for WL, however, differs in that in USSAERO the term G_2 should be replaced by $G_2 - G_3$. (Because the function G_3 comes in the form λG_3, it does not cancel in the superposition of the semi-infinite regions if the panel is tapered.)

The error in USSAERO is not present if $z = 0$ (points in the wing plane) and is small for small z (points close to the wing plane). This was precisely the situation in the earlier versions of USSAERO, and it offers an explanation as to why the error has not been detected. However, with extensions to USSAERO for external flowfield predictions and even external stores, the values of z become large enough for the error to be significant.

Figure 6 shows the vertical flowfield perturbation velocities along line l below the wing, using the WL expression of Eqs. (8). It is seen that the severe dependence of the vertical velocities on the chordwise paneling, as observed in Fig. 2, has now disappeared.

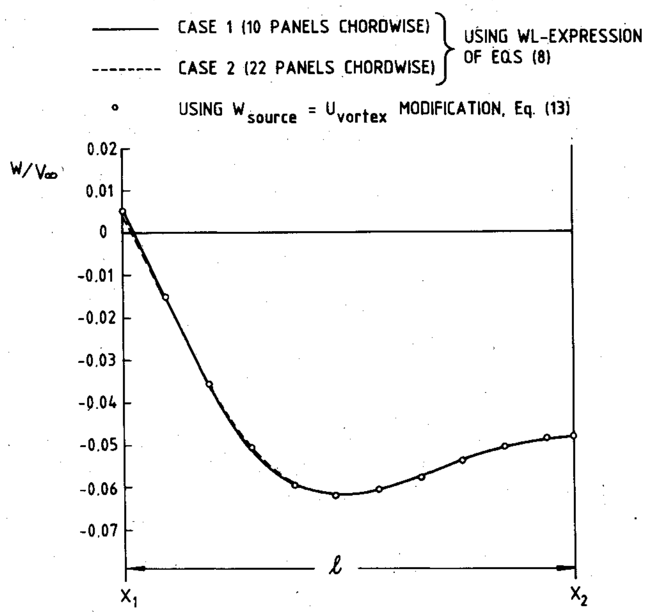

Fig. 6 The vertical perturbation velocities along line l using the corrected USSAERO code ($M_\infty = 0.4$, $\alpha = 6$ deg).

As $G_3 = 0$ in supersonic flow, the above error in USSAERO vanishes in supersonic flow and the calculation of the vertical velocity components should not present problems. This is confirmed in Fig. 7, where the test cases of Fig. 2 were run for $M_\infty = 1.4$, using the uncorrected USSAERO code.

It will be realized that with the corrections to USSAERO as outlined above, there will still be a residual source sheet behind a tapered wing panel. Thus, the vertical perturbation velocity components induced by the thickness of a tapered wing are discontinuous across the wing plane behind the wing trailing edge, whereas they should be zero in this plane. This may present problems, for example, in the case of a horizontal tail which lies in the plane of the wing. Lift may then be generated in situations where the lift should actually be zero. These undesirable effects of the residual source sheet upon the vertical perturbation velocities can be removed by treating the linear source distribution on a wing panel in exactly the same manner as the linear vortex distribution on a wing panel has been treated in USSAERO (Refs. 1 and 4). The line sources are then placed on the radial lines through the intersection of the leading and trailing edges of the panel, and integration takes place from the leading edge to the trailing edge over the radial lines. Using the same manipulations done in USSAERO for the linear vortex distribution on a wing panel, the linear source distribution is made constant along the radial lines, so that it is linear in chordwise direction only, see Fig. 8. Leaving the mathematical analysis aside, it can be verified that this procedure leads to the result that the vertical perturbation velocity component induced by the unit linear panel source distribution just described (w_{source}) is equal to the axial perturbation velocity component induced by the unit linear panel vortex distribution as calculated in USSAERO

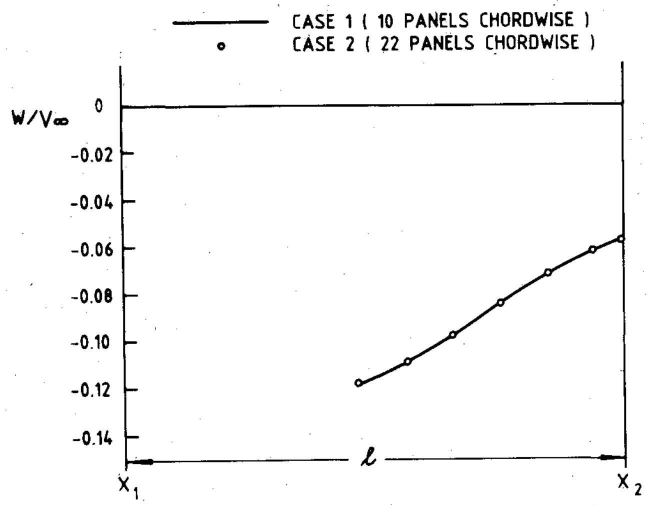

Fig. 7 The vertical perturbation velocities along line l using the uncorrected USSAERO code ($M_\infty = 1.4$, $\alpha = 6$ deg).

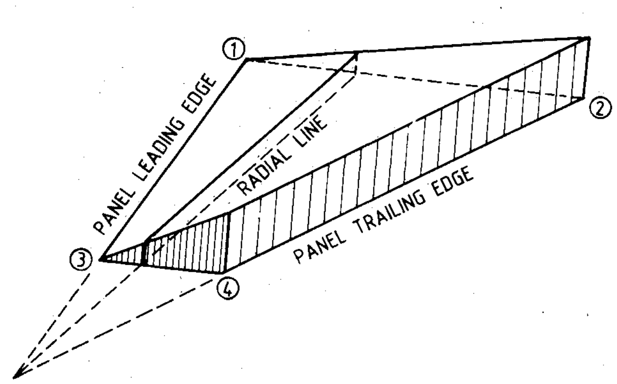

Fig. 8 Linear wing panel source distribution (constant along radial lines).

(u_{vortex}), i.e.,

$$w_{\text{source}} = u_{\text{vortex}} \tag{13}$$

It is readily verified that the vertical perturbation velocity component in the panel plane is now everywhere zero outside the panel.

The vertical perturbation velocity components along line l below the wing as obtained using modification (13) is also shown in Fig. 6. The results are nearly identical to those obtained using the *WL* expression of Eqs. (8). Reference 5 gives several comparisons of flowfields, as calculated with the modified vertical velocities described above, with experimental data.

Concluding Remarks

Two remedies have been presented to correct a basic error in the various versions of the USSAERO computer code. The updates required in USSAERO to effect these remedies can be provided by the author upon request.

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